# Introduction to the Si3D flow model
## (Prepared by Fabián A. Bombardelli; adapted from McDonald, MS Thesis, UC Davis, 2007)

Smith (1997) authored a semi-implicit, three-dimensional, finite-difference model for estuarine circulation in the FORTRAN 90 programming language. This program, commonly known as Si3D, was implemented to simulate circulation in the San Francisco Bay and Estuary from the ocean to the delta. The Si3D program simulates estuarine flows within a three-dimensional grid which is horizontally resolved by squares and vertically resolved by layers. A bathymetry file identifies which cells are open to flows and which ones are closed. A salinity initial condition is needed, and boundary files are used to indicate water surface elevation, flow rate and salinity, if necessary, for every time increment at each open boundary at the model perimeter. All modeling parameters and output specifications are given in the general input file.

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}-fv=-\frac{1}{\rho_0}\frac{\partial \rho}{\partial x}+\frac{\partial}{\partial x}\left(A_H\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial u}{\partial z}\right) \tag{2}$$

$$\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial vv}{\partial y}+\frac{\partial vw}{\partial z}+fu=-\frac{1}{\rho_0}\frac{\partial \rho}{\partial y}+\frac{\partial}{\partial x}\left(A_H\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial v}{\partial z}\right) \tag{3}$$

$$0=-\frac{1}{\rho_0}\frac{\partial \rho}{\partial z}-\frac{\rho}{\rho_0}g \tag{4}$$

$$\frac{\partial s}{\partial t}+\frac{\partial us}{\partial x}+\frac{\partial vs}{\partial y}+\frac{\partial ws}{\partial z}=\frac{\partial}{\partial x}\left(D_H\frac{\partial s}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_H\frac{\partial s}{\partial y}\right)+\frac{\partial}{\partial z}\left(D_V\frac{\partial s}{\partial z}\right) \tag{5}$$

where *u*, *v* and *w* are the velocities in the *x*, *y* and *z* directions, respectively, and *f* is the Coriolis parameter. The advective acceleration terms, $\partial uu/\partial x$, $\partial uv/\partial y$, $\partial uw/\partial z$, $\partial uv/\partial x$, $\partial vv/\partial y$, and $\partial vw/\partial z$, are written in a conservative or divergence form.

By substituting in baroclinic terms for the pressure gradient terms in the *x*- and *y*-momentum equations, assuming $\rho_s = \rho_0$ to the same order of approximation as the Boussinesq approximation, we obtain a form that applies to estuarine tidal flows influenced by density variations.

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}-fv=$$
$$-g\frac{\partial \zeta}{\partial x}-g\frac{1}{\rho_0}\int_z^{\zeta}\frac{\partial \rho}{\partial x}dz'+\frac{\partial}{\partial x}\left(A_H\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial u}{\partial z}\right) \tag{6}$$

$$\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial vv}{\partial y}+\frac{\partial vw}{\partial z}+fu=$$
$$-g\frac{\partial \zeta}{\partial y}-g\frac{1}{\rho_0}\int_z^{\zeta}\frac{\partial \rho}{\partial x}dz'+\frac{\partial}{\partial x}\left(A_H\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial v}{\partial z}\right) \tag{7}$$

The layer-averaged form of the Si3D governing equations is as follows.

Continuity equations:

$$\frac{\partial \zeta}{\partial t}+\frac{\partial}{\partial x}\left(\sum_{k=1}^{km}U_k\right)+\frac{\partial}{\partial y}\left(\sum_{k=1}^{km}V_k\right)=0 \tag{8}$$

$$\left(w\right)\Big|_{k+½}^{k-½}=-\frac{\partial U_k}{\partial x}-\frac{\partial V_k}{\partial y}\qquad k=2,3,\ldots,km \tag{9}$$

Momentum equations:

$$\frac{\partial U_k}{\partial t}+\frac{\partial (Uu)_k}{\partial x}+\frac{\partial (Vu)_k}{\partial y}+\left(uw\right)\Big|_{k+½}^{k-½}-fV_k+\frac{h_k}{\rho_k}g\rho_1\frac{\partial \zeta}{\partial x}= \tag{10}$$
$$-\frac{h_k}{\rho_k}\left[\frac{gh_1}{2}\frac{\partial \rho_1}{\partial x}+\sum_{m=2}^{k}\left(\frac{gh_{m-1}}{2}\frac{\partial \rho_{m-1}}{\partial x}+\frac{gh_m}{2}\frac{\partial \rho_m}{\partial x}\right)\right]+\frac{\partial}{\partial x}\left(A_H h\frac{\partial u}{\partial x}\right)_k+\frac{\partial}{\partial y}\left(A_H h\frac{\partial u}{\partial y}\right)_k+\left(\frac{\tau_{xz}}{\rho}\right)\Bigg|_{k+½}^{k-½}$$

$$\frac{\partial V_k}{\partial t}+\frac{\partial (Uv)_k}{\partial x}+\frac{\partial (Vv)_k}{\partial y}+\left(vw\right)\Big|_{k+½}^{k-½}+fU_k+\frac{h_k}{\rho_k}g\rho_1\frac{\partial \zeta}{\partial y}= \tag{11}$$
$$-\frac{h_k}{\rho_k}\left[\frac{gh_1}{2}\frac{\partial \rho_1}{\partial y}+\sum_{m=2}^{k}\left(\frac{gh_{m-1}}{2}\frac{\partial \rho_{m-1}}{\partial y}+\frac{gh_m}{2}\frac{\partial \rho_m}{\partial y}\right)\right]+\frac{\partial}{\partial x}\left(A_H h\frac{\partial v}{\partial x}\right)_k+\frac{\partial}{\partial y}\left(A_H h\frac{\partial v}{\partial y}\right)_k+\left(\frac{\tau_{yz}}{\rho}\right)\Bigg|_{k+½}^{k-½}$$

Salt transport equation:

$$\frac{\partial (hs)_k}{\partial t}+\frac{\partial (uhs)_k}{\partial x}+\frac{\partial (vhs)_k}{\partial y}+\left(ws\right)\Big|_{k+½}^{k-½}=\frac{\partial}{\partial x}\left(D_H h\frac{\partial s}{\partial x}\right)_k+\frac{\partial}{\partial y}\left(D_H h\frac{\partial s}{\partial y}\right)_k+\left(\frac{J_z}{\rho}\right)\Bigg|_{k+½}^{k-½} \tag{12}$$

The notation $\left(\ \right)\Big|_{k+½}^{k-½}$ represents the difference between interface values for a particular layer. Layer-averaged density $\rho_k$ has been substituted for $\rho_0$ in the denominator of the pressure, vertical stress and vertical salt flux terms. This substitution reduces any error caused by the Boussinesq approximation. The bottom and free surface boundary conditions are satisfied by defining $w_{km+½}=0$, $(uw)_{½}=(uw)_{km+½}=0$, $(vw)_{½}=(vw)_{km+½}=0$, $(\tau_{xz},\tau_{yz})_{½}=(\tau_{xs},\tau_{ys})$, $(\tau_{xz},\tau_{yz})_{km+½}=(\tau_{xb},\tau_{yb})$, $(ws)_{½}=(ws)_{km+½}=0$ and $(J_z)_{½}=(J_z)_{km+½}=0$. The summation term in Equations 10 and 11 is omitted for a surface layer ($k$ = 1). These five three-dimensional governing equations are discretized using semi-implicit leapfrog and semi-implicit trapezoidal finite-differencing schemes.

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$$-\frac{h_k}{\rho_k}\left[\frac{gh_1}{2}\frac{\partial \rho_1}{\partial x}+\sum_{m=2}^{k}\left(\frac{gh_{m-1}}{2}\frac{\partial \rho_{m-1}}{\partial x}+\frac{gh_m}{2}\frac{\partial \rho_m}{\partial x}\right)\right]+\frac{\partial}{\partial x}\left(A_H h\frac{\partial u}{\partial x}\right)_k+\frac{\partial}{\partial y}\left(A_H h\frac{\partial u}{\partial y}\right)_k+\left(\frac{\tau_{xz}}{\rho}\right)\Bigg|_{k+½}^{k-½}$$

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Continuity equations:

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$$-\frac{h_k}{\rho_k}\left[\frac{gh_1}{2}\frac{\partial \rho_1}{\partial x}+\sum_{m=2}^{k}\left(\frac{gh_{m-1}}{2}\frac{\partial \rho_{m-1}}{\partial x}+\frac{gh_m}{2}\frac{\partial \rho_m}{\partial x}\right)\right]+\frac{\partial}{\partial x}\left(A_H h\frac{\partial u}{\partial x}\right)_k+\frac{\partial}{\partial y}\left(A_H h\frac{\partial u}{\partial y}\right)_k+\left(\frac{\tau_{xz}}{\rho}\right)\Bigg|_{k+½}^{k-½}$$

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}-fv=-\frac{1}{\rho_0}\frac{\partial \rho}{\partial x}+\frac{\partial}{\partial x}\left(A_H\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial u}{\partial z}\right) \tag{2}$$

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where *u*, *v* and *w* are the velocities in the *x*, *y* and *z* directions, respectively, and *f* is the Coriolis parameter. The advective acceleration terms, $\partial uu/\partial x, \partial uv/\partial y, \partial uw/\partial z, \partial uv/\partial x, \partial vv/\partial y, \text{and} \, \partial vw/\partial z,$ are written in a conservative or divergence form.

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Smith (1997) authored a semi-implicit, three-dimensional, finite-difference model for estuarine circulation in the FORTRAN 90 programming language. This program, commonly known as Si3D, was implemented to simulate circulation in the San Francisco Bay and Estuary from the ocean to the delta. The Si3D program simulates estuarine flows within a three-dimensional grid which is horizontally resolved by squares and vertically resolved by layers. A bathymetry file identifies which cells are open to flows and which ones are closed. A salinity initial condition is needed, and boundary files are used to indicate water surface elevation, flow rate and salinity, if necessary, for every time increment at each open boundary at the model perimeter. All modeling parameters and output specifications are given in the general input file.

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}-fv=-\frac{1}{\rho_0}\frac{\partial \rho}{\partial x}+\frac{\partial}{\partial x}\left(A_H\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial u}{\partial z}\right) \tag{2}$$

$$\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial vv}{\partial y}+\frac{\partial vw}{\partial z}+fu=-\frac{1}{\rho_0}\frac{\partial \rho}{\partial y}+\frac{\partial}{\partial x}\left(A_H\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial v}{\partial z}\right) \tag{3}$$

$$0=-\frac{1}{\rho_0}\frac{\partial \rho}{\partial z}-\frac{\rho}{\rho_0}g \tag{4}$$

$$\frac{\partial s}{\partial t}+\frac{\partial us}{\partial x}+\frac{\partial vs}{\partial y}+\frac{\partial ws}{\partial z}=\frac{\partial}{\partial x}\left(D_H\frac{\partial s}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_H\frac{\partial s}{\partial y}\right)+\frac{\partial}{\partial z}\left(D_V\frac{\partial s}{\partial z}\right) \tag{5}$$

where $u$, $v$ and $w$ are the velocities in the $x$, $y$ and $z$ directions, respectively, and $f$ is the Coriolis parameter. The advective acceleration terms, $\partial uu/\partial x$, $\partial uv/\partial y$, $\partial uw/\partial z$, $\partial uv/\partial x$, $\partial vv/\partial y$, and $\partial vw/\partial z$, are written in a conservative or divergence form.

By substituting in baroclinic terms for the pressure gradient terms in the *x*- and *y*-momentum equations, assuming $\rho_s = \rho_0$ to the same order of approximation as the Boussinesq approximation, we obtain a form that applies to estuarine tidal flows influenced by density variations.

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}-fv=$$

$$-g\frac{\partial \zeta}{\partial x}-g\frac{1}{\rho_0}\int_z^{\zeta}\frac{\partial \rho}{\partial x}dz'+\frac{\partial}{\partial x}\left(A_H\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial u}{\partial z}\right) \tag{6}$$

$$\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial vv}{\partial y}+\frac{\partial vw}{\partial z}+fu=$$

$$-g\frac{\partial \zeta}{\partial y}-g\frac{1}{\rho_0}\int_z^{\zeta}\frac{\partial \rho}{\partial x}dz'+\frac{\partial}{\partial x}\left(A_H\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial v}{\partial z}\right) \tag{7}$$

The layer-averaged form of the Si3D governing equations is as follows.
Continuity equations:

$$\frac{\partial \zeta}{\partial t}+\frac{\partial}{\partial x}\left(\sum_{k=1}^{km}U_k\right)+\frac{\partial}{\partial y}\left(\sum_{k=1}^{km}V_k\right)=0 \tag{8}$$

$$\left(w\right)\Big|_{k+½}^{k-½}=-\frac{\partial U_k}{\partial x}-\frac{\partial V_k}{\partial y}\quad k=2,\,3,\,\ldots,\,km \tag{9}$$

Momentum equations:

$$\frac{\partial U_k}{\partial t}+\frac{\partial (Uu)_k}{\partial x}+\frac{\partial (Vu)_k}{\partial y}+\left(uw\right)\Big|_{k+½}^{k-½}-fV_k+\frac{h_k}{\rho_k}g\rho_1\frac{\partial \zeta}{\partial x}= \tag{10}$$

$$-\frac{h_k}{\rho_k}\left[\frac{gh_1}{2}\frac{\partial \rho_1}{\partial x}+\sum_{m=2}^{k}\left(\frac{gh_{m-1}}{2}\frac{\partial \rho_{m-1}}{\partial x}+\frac{gh_m}{2}\frac{\partial \rho_m}{\partial x}\right)\right]+\frac{\partial}{\partial x}\left(A_H h\frac{\partial u}{\partial x}\right)_k+\frac{\partial}{\partial y}\left(A_H h\frac{\partial u}{\partial y}\right)_k+\left(\frac{\tau_{xz}}{\rho}\right)\Bigg|_{k+½}^{k-½}$$

$$\frac{\partial V_k}{\partial t}+\frac{\partial (Uv)_k}{\partial x}+\frac{\partial (Vv)_k}{\partial y}+\left(vw\right)\Big|_{k+½}^{k-½}+fU_k+\frac{h_k}{\rho_k}g\rho_1\frac{\partial \zeta}{\partial y}= \tag{11}$$

$$-\frac{h_k}{\rho_k}\left[\frac{gh_1}{2}\frac{\partial \rho_1}{\partial y}+\sum_{m=2}^{k}\left(\frac{gh_{m-1}}{2}\frac{\partial \rho_{m-1}}{\partial y}+\frac{gh_m}{2}\frac{\partial \rho_m}{\partial y}\right)\right]+\frac{\partial}{\partial x}\left(A_H h\frac{\partial v}{\partial x}\right)_k+\frac{\partial}{\partial y}\left(A_H h\frac{\partial v}{\partial y}\right)_k+\left(\frac{\tau_{yz}}{\rho}\right)\Bigg|_{k+½}^{k-½}$$

Salt transport equation:

$$\frac{\partial (hs)_k}{\partial t}+\frac{\partial (uhs)_k}{\partial x}+\frac{\partial (vhs)_k}{\partial y}+\left(ws\right)\Big|_{k+½}^{k-½}=\frac{\partial}{\partial x}\left(D_H h\frac{\partial s}{\partial x}\right)_k+\frac{\partial}{\partial y}\left(D_H h\frac{\partial s}{\partial y}\right)_k+\left(\frac{J_z}{\rho}\right)\Bigg|_{k+½}^{k-½} \tag{12}$$

The notation $(\;)\Big|_{k+½}^{k-½}$ represents the difference between interface values for a particular layer. Layer-averaged density $\rho_k$ has been substituted for $\rho_0$ in the denominator of the pressure, vertical stress and vertical salt flux terms. This substitution reduces any error caused by the Boussinesq approximation. The bottom and free surface boundary conditions are satisfied by defining $w_{km+½} = 0$, $(uw)_{½} = (uw)_{km+½} = 0$, $(vw)_{½} = (vw)_{km+½} = 0$, $(\tau_{xz}, \tau_{yz})_{½} = (\tau_{xs}, \tau_{ys})$, $(\tau_{xz}, \tau_{yz})_{km+½} = (\tau_{xb}, \tau_{yb})$, $(ws)_{½} = (ws)_{km+½} = 0$ and $(J_z)_{½} = (J_z)_{km+½} = 0$. The summation term in Equations 10 and 11 is omitted for a surface layer ($k = 1$). These five three-dimensional governing equations are discretized using semi-implicit leapfrog and semi-implicit trapezoidal finite-differencing schemes.

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$$\begin{aligned}&\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial vv}{\partial y}+\frac{\partial vw}{\partial z}+fu=\\&\qquad -g\frac{\partial \zeta}{\partial y}-g\frac{1}{\rho_0}\int_z^{\zeta}\frac{\partial \rho}{\partial x}dz'+\frac{\partial}{\partial x}\left(A_H\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial v}{\partial z}\right)\end{aligned}\tag{7}$$

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

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where $u$, $v$ and $w$ are the velocities in the $x$, $y$ and $z$ directions, respectively, and $f$ is the Coriolis parameter. The advective acceleration terms, $\partial uu/\partial x$, $\partial uv/\partial y$, $\partial uw/\partial z$, $\partial uv/\partial x$, $\partial vv/\partial y$, and $\partial vw/\partial z$, are written in a conservative or divergence form.

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Salt transport equation:

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}-fv=-\frac{1}{\rho_0}\frac{\partial \rho}{\partial x}+\frac{\partial}{\partial x}\left(A_H\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial u}{\partial z}\right) \tag{2}$$

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$$\frac{\partial s}{\partial t}+\frac{\partial us}{\partial x}+\frac{\partial vs}{\partial y}+\frac{\partial ws}{\partial z}=\frac{\partial}{\partial x}\left(D_H\frac{\partial s}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_H\frac{\partial s}{\partial y}\right)+\frac{\partial}{\partial z}\left(D_V\frac{\partial s}{\partial z}\right) \tag{5}$$

where $u$, $v$ and $w$ are the velocities in the $x$, $y$ and $z$ directions, respectively, and $f$ is the Coriolis parameter. The advective acceleration terms, $\partial uu/\partial x,\ \partial uv/\partial y,\ \partial uw/\partial z,$ $\partial uv/\partial x,\ \partial vv/\partial y,\text{ and}\,\partial vw/\partial z,$ are written in a conservative or divergence form.

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Continuity equations:

$$\frac{\partial \zeta}{\partial t}+\frac{\partial}{\partial x}\left(\sum_{k=1}^{km}U_k\right)+\frac{\partial}{\partial y}\left(\sum_{k=1}^{km}V_k\right)=0 \tag{8}$$

$$\left.(w)\right|_{k+½}^{k-½}=-\frac{\partial U_k}{\partial x}-\frac{\partial V_k}{\partial y}\qquad k=2,3,\ldots,km \tag{9}$$

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$$\frac{\partial \zeta}{\partial t}+\frac{\partial}{\partial x}\left(\sum_{k=1}^{km}U_k\right)+\frac{\partial}{\partial y}\left(\sum_{k=1}^{km}V_k\right)=0 \tag{8}$$

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The notation $(\ )\big|_{k+½}^{k-½}$ represents the difference between interface values for a particular layer. Layer-averaged density $\rho_k$ has been substituted for $\rho_0$ in the denominator of the pressure, vertical stress and vertical salt flux terms. This substitution reduces any error caused by the Boussinesq approximation. The bottom and free surface boundary conditions are satisfied by defining $w_{km+½} = 0$, $(uw)_{½} = (uw)_{km+½} = 0$, $(vw)_{½} = (vw)_{km+½} = 0$, $(\tau_{xz}, \tau_{yz})_{½} = (\tau_{xs}, \tau_{ys})$, $(\tau_{xz}, \tau_{yz})_{km+½} = (\tau_{xb}, \tau_{yb})$, $(ws)_{½} = (ws)_{km+½} = 0$ and $(J_z)_{½} = (J_z)_{km+½} = 0$. The summation term in Equations 10 and 11 is omitted for a surface layer ($k = 1$). These five three-dimensional governing equations are discretized using semi-implicit leapfrog and semi-implicit trapezoidal finite-differencing schemes.

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$$-\frac{h_k}{\rho_k}\left[\frac{gh_1}{2}\frac{\partial \rho_1}{\partial x}+\sum_{m=2}^{k}\left(\frac{gh_{m-1}}{2}\frac{\partial \rho_{m-1}}{\partial x}+\frac{gh_m}{2}\frac{\partial \rho_m}{\partial x}\right)\right]+\frac{\partial}{\partial x}\left(A_H h\frac{\partial u}{\partial x}\right)_k+\frac{\partial}{\partial y}\left(A_H h\frac{\partial u}{\partial y}\right)_k+\left(\frac{\tau_{xz}}{\rho}\right)\Bigg|_{k+½}^{k-½}$$

$$\frac{\partial V_k}{\partial t}+\frac{\partial (Uv)_k}{\partial x}+\frac{\partial (Vv)_k}{\partial y}+\left(vw\right)\Big|_{k+½}^{k-½}+fU_k+\frac{h_k}{\rho_k}g\rho_1\frac{\partial \zeta}{\partial y}= \tag{11}$$
$$-\frac{h_k}{\rho_k}\left[\frac{gh_1}{2}\frac{\partial \rho_1}{\partial y}+\sum_{m=2}^{k}\left(\frac{gh_{m-1}}{2}\frac{\partial \rho_{m-1}}{\partial y}+\frac{gh_m}{2}\frac{\partial \rho_m}{\partial y}\right)\right]+\frac{\partial}{\partial x}\left(A_H h\frac{\partial v}{\partial x}\right)_k+\frac{\partial}{\partial y}\left(A_H h\frac{\partial v}{\partial y}\right)_k+\left(\frac{\tau_{yz}}{\rho}\right)\Bigg|_{k+½}^{k-½}$$

Salt transport equation:

$$\frac{\partial (hs)_k}{\partial t}+\frac{\partial (uhs)_k}{\partial x}+\frac{\partial (vhs)_k}{\partial y}+\left(ws\right)\Big|_{k+½}^{k-½}=\frac{\partial}{\partial x}\left(D_H h\frac{\partial s}{\partial x}\right)_k+\frac{\partial}{\partial y}\left(D_H h\frac{\partial s}{\partial y}\right)_k+\left(\frac{J_z}{\rho}\right)\Bigg|_{k+½}^{k-½} \tag{12}$$

The notation $\left(\;\right)\Big|_{k+½}^{k-½}$ represents the difference between interface values for a particular layer. Layer-averaged density $\rho_k$ has been substituted for $\rho_0$ in the denominator of the pressure, vertical stress and vertical salt flux terms. This substitution reduces any error caused by the Boussinesq approximation. The bottom and free surface boundary conditions are satisfied by defining $w_{km+½}=0$, $(uw)_{½}=(uw)_{km+½}=0$, $(vw)_{½}=(vw)_{km+½}=0$, $(\tau_{xz},\ \tau_{yz})_{½}=(\tau_{xs},\ \tau_{ys})$, $(\tau_{xz},\ \tau_{yz})_{km+½}=(\tau_{xb},\ \tau_{yb})$, $(ws)_{½}=(ws)_{km+½}=0$ and $(J_z)_{½}=(J_z)_{km+½}=0$. The summation term in Equations 10 and 11 is omitted for a surface layer ($k=1$). These five three-dimensional governing equations are discretized using semi-implicit leapfrog and semi-implicit trapezoidal finite-differencing schemes.

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The shallow-water flows of an estuary can essentially be considered horizontal and, therefore, vertical velocities and accelerations are negligible compared to gravity. As a result of this assumption, only the pressure and gravity terms are retained in the *z*-momentum equation, reducing it to the hydrostatic pressure equation. The Coriolis terms in the horizontal momentum equations involving vertical velocity w can also be neglected. By incorporating these assumptions, the continuity, *x*-momentum, *y*-momentum, *z*-momentum and salt transport equations become the following five equations, respectively:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}-fv=-\frac{1}{\rho_0}\frac{\partial \rho}{\partial x}+\frac{\partial}{\partial x}\left(A_H\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial u}{\partial z}\right) \tag{2}$$

$$\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial vv}{\partial y}+\frac{\partial vw}{\partial z}+fu=-\frac{1}{\rho_0}\frac{\partial \rho}{\partial y}+\frac{\partial}{\partial x}\left(A_H\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial v}{\partial z}\right) \tag{3}$$

$$0=-\frac{1}{\rho_0}\frac{\partial \rho}{\partial z}-\frac{\rho}{\rho_0}g \tag{4}$$

$$\frac{\partial s}{\partial t}+\frac{\partial us}{\partial x}+\frac{\partial vs}{\partial y}+\frac{\partial ws}{\partial z}=\frac{\partial}{\partial x}\left(D_H\frac{\partial s}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_H\frac{\partial s}{\partial y}\right)+\frac{\partial}{\partial z}\left(D_V\frac{\partial s}{\partial z}\right) \tag{5}$$

where $u$, $v$ and $w$ are the velocities in the $x$, $y$ and $z$ directions, respectively, and $f$ is the Coriolis parameter. The advective acceleration terms, $\partial uu/\partial x$, $\partial uv/\partial y$, $\partial uw/\partial z$, $\partial uv/\partial x$, $\partial vv/\partial y$, and $\partial vw/\partial z$, are written in a conservative or divergence form.

By substituting in baroclinic terms for the pressure gradient terms in the *x*- and *y*-momentum equations, assuming $\rho_s = \rho_0$ to the same order of approximation as the Boussinesq approximation, we obtain a form that applies to estuarine tidal flows influenced by density variations.

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}-fv=$$
$$-g\frac{\partial \zeta}{\partial x}-g\frac{1}{\rho_0}\int_z^{\zeta}\frac{\partial \rho}{\partial x}dz'+\frac{\partial}{\partial x}\left(A_H\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial u}{\partial z}\right) \tag{6}$$

$$\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial vv}{\partial y}+\frac{\partial vw}{\partial z}+fu=$$
$$-g\frac{\partial \zeta}{\partial y}-g\frac{1}{\rho_0}\int_z^{\zeta}\frac{\partial \rho}{\partial x}dz'+\frac{\partial}{\partial x}\left(A_H\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial v}{\partial z}\right) \tag{7}$$

The layer-averaged form of the Si3D governing equations is as follows.
Continuity equations:

$$\frac{\partial \zeta}{\partial t}+\frac{\partial}{\partial x}\left(\sum_{k=1}^{km}U_k\right)+\frac{\partial}{\partial y}\left(\sum_{k=1}^{km}V_k\right)=0 \tag{8}$$

$$\left(w\right)\Big|_{k+\frac{1}{2}}^{k-\frac{1}{2}}=-\frac{\partial U_k}{\partial x}-\frac{\partial V_k}{\partial y}\qquad k=2,\,3,\,.\,.\,.\,,\,km \tag{9}$$

Momentum equations:

$$\frac{\partial U_k}{\partial t}+\frac{\partial (Uu)_k}{\partial x}+\frac{\partial (Vu)_k}{\partial y}+\left(uw\right)\Big|_{k+\frac{1}{2}}^{k-\frac{1}{2}}-fV_k+\frac{h_k}{\rho_k}g\rho_1\frac{\partial \zeta}{\partial x}= \tag{10}$$
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Salt transport equation:

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The notation $(\ )\Big|_{k+\frac{1}{2}}^{k-\frac{1}{2}}$ represents the difference between interface values for a particular layer. Layer-averaged density $\rho_k$ has been substituted for $\rho_0$ in the denominator of the pressure, vertical stress and vertical salt flux terms. This substitution reduces any error caused by the Boussinesq approximation. The bottom and free surface boundary conditions are satisfied by defining $w_{km+\frac{1}{2}}=0$, $(uw)_{\frac{1}{2}}=(uw)_{km+\frac{1}{2}}=0$, $(vw)_{\frac{1}{2}}=(vw)_{km+\frac{1}{2}}=0$, $(\tau_{xz},\ \tau_{yz})_{\frac{1}{2}}=(\tau_{xs},\ \tau_{ys})$, $(\tau_{xz},\ \tau_{yz})_{km+\frac{1}{2}}=(\tau_{xb},\ \tau_{yb})$, $(ws)_{\frac{1}{2}}=(ws)_{km+\frac{1}{2}}=0$ and $(J_z)_{\frac{1}{2}}=(J_z)_{km+\frac{1}{2}}=0$. The summation term in Equations 10 and 11 is omitted for a surface layer ($k$ = 1). These five three-dimensional governing equations are discretized using semi-implicit leapfrog and semi-implicit trapezoidal finite-differencing schemes.

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$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}-fv=-\frac{1}{\rho_0}\frac{\partial \rho}{\partial x}+\frac{\partial}{\partial x}\left(A_H\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial u}{\partial z}\right) \tag{2}$$

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$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}-fv=$$

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Continuity equations:

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Momentum equations:

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Salt transport equation:

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$$\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial vv}{\partial y}+\frac{\partial vw}{\partial z}+fu=$$
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Salt transport equation:

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# (Prepared by Fabián A. Bombardelli; adapted from McDonald, MS Thesis, UC Davis, 2007)

Smith (1997) authored a semi-implicit, three-dimensional, finite-difference model for estuarine circulation in the FORTRAN 90 programming language. This program, commonly known as Si3D, was implemented to simulate circulation in the San Francisco Bay and Estuary from the ocean to the delta. The Si3D program simulates estuarine flows within a three-dimensional grid which is horizontally resolved by squares and vertically resolved by layers. A bathymetry file identifies which cells are open to flows and which ones are closed. A salinity initial condition is needed, and boundary files are used to indicate water surface elevation, flow rate and salinity, if necessary, for every time increment at each open boundary at the model perimeter. All modeling parameters and output specifications are given in the general input file.

The shallow-water flows of an estuary can essentially be considered horizontal and, therefore, vertical velocities and accelerations are negligible compared to gravity. As a result of this assumption, only the pressure and gravity terms are retained in the *z*-momentum equation, reducing it to the hydrostatic pressure equation. The Coriolis terms in the horizontal momentum equations involving vertical velocity w can also be neglected. By incorporating these assumptions, the continuity, *x*-momentum, *y*-momentum, *z*-momentum and salt transport equations become the following five equations, respectively:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}-fv=-\frac{1}{\rho_0}\frac{\partial \rho}{\partial x}+\frac{\partial}{\partial x}\left(A_H\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial u}{\partial z}\right) \tag{2}$$

$$\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial vv}{\partial y}+\frac{\partial vw}{\partial z}+fu=-\frac{1}{\rho_0}\frac{\partial \rho}{\partial y}+\frac{\partial}{\partial x}\left(A_H\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial v}{\partial z}\right) \tag{3}$$

$$0=-\frac{1}{\rho_0}\frac{\partial \rho}{\partial z}-\frac{\rho}{\rho_0}g \tag{4}$$

$$\frac{\partial s}{\partial t}+\frac{\partial us}{\partial x}+\frac{\partial vs}{\partial y}+\frac{\partial ws}{\partial z}=\frac{\partial}{\partial x}\left(D_H\frac{\partial s}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_H\frac{\partial s}{\partial y}\right)+\frac{\partial}{\partial z}\left(D_V\frac{\partial s}{\partial z}\right) \tag{5}$$

where *u*, *v* and *w* are the velocities in the *x*, *y* and *z* directions, respectively, and *f* is the Coriolis parameter. The advective acceleration terms, $\partial uu/\partial x$, $\partial uv/\partial y$, $\partial uw/\partial z$, $\partial uv/\partial x$, $\partial vv/\partial y$, and $\partial vw/\partial z$, are written in a conservative or divergence form.

By substituting in baroclinic terms for the pressure gradient terms in the *x*- and *y*-momentum equations, assuming $\rho_s = \rho_0$ to the same order of approximation as the Boussinesq approximation, we obtain a form that applies to estuarine tidal flows influenced by density variations.

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}-fv= -g\frac{\partial \zeta}{\partial x}-g\frac{1}{\rho_0}\int_z^{\zeta}\frac{\partial \rho}{\partial x}dz'+\frac{\partial}{\partial x}\left(A_H\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial u}{\partial z}\right) \tag{6}$$

$$\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial vv}{\partial y}+\frac{\partial vw}{\partial z}+fu= -g\frac{\partial \zeta}{\partial y}-g\frac{1}{\rho_0}\int_z^{\zeta}\frac{\partial \rho}{\partial x}dz'+\frac{\partial}{\partial x}\left(A_H\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_H\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_V\frac{\partial v}{\partial z}\right) \tag{7}$$

The layer-averaged form of the Si3D governing equations is as follows.
Continuity equations:

$$\frac{\partial \zeta}{\partial t}+\frac{\partial}{\partial x}\left(\sum_{k=1}^{km}U_k\right)+\frac{\partial}{\partial y}\left(\sum_{k=1}^{km}V_k\right)=0 \tag{8}$$

$$\left(w\right)\Big|_{k+½}^{k-½}=-\frac{\partial U_k}{\partial x}-\frac{\partial V_k}{\partial y} \quad k=2,3,\ldots,km \tag{9}$$

Momentum equations:

$$\frac{\partial U_k}{\partial t}+\frac{\partial (Uu)_k}{\partial x}+\frac{\partial (Vu)_k}{\partial y}+\left(uw\right)\Big|_{k+½}^{k-½}-fV_k+\frac{h_k}{\rho_k}g\rho_1\frac{\partial \zeta}{\partial x}= -\frac{h_k}{\rho_k}\left[\frac{gh_1}{2}\frac{\partial \rho_1}{\partial x}+\sum_{m=2}^{k}\left(\frac{gh_{m-1}}{2}\frac{\partial \rho_{m-1}}{\partial x}+\frac{gh_m}{2}\frac{\partial \rho_m}{\partial x}\right)\right]+\frac{\partial}{\partial x}\left(A_H h\frac{\partial u}{\partial x}\right)_k+\frac{\partial}{\partial y}\left(A_H h\frac{\partial u}{\partial y}\right)_k+\left(\frac{\tau_{xz}}{\rho}\right)\Bigg|_{k+½}^{k-½} \tag{10}$$

$$\frac{\partial V_k}{\partial t}+\frac{\partial (Uv)_k}{\partial x}+\frac{\partial (Vv)_k}{\partial y}+\left(vw\right)\Big|_{k+½}^{k-½}+fU_k+\frac{h_k}{\rho_k}g\rho_1\frac{\partial \zeta}{\partial y}= -\frac{h_k}{\rho_k}\left[\frac{gh_1}{2}\frac{\partial \rho_1}{\partial y}+\sum_{m=2}^{k}\left(\frac{gh_{m-1}}{2}\frac{\partial \rho_{m-1}}{\partial y}+\frac{gh_m}{2}\frac{\partial \rho_m}{\partial y}\right)\right]+\frac{\partial}{\partial x}\left(A_H h\frac{\partial v}{\partial x}\right)_k+\frac{\partial}{\partial y}\left(A_H h\frac{\partial v}{\partial y}\right)_k+\left(\frac{\tau_{yz}}{\rho}\right)\Bigg|_{k+½}^{k-½} \tag{11}$$

Salt transport equation:

$$\frac{\partial (hs)_k}{\partial t}+\frac{\partial (uhs)_k}{\partial x}+\frac{\partial (vhs)_k}{\partial y}+\left(ws\right)\Big|_{k+½}^{k-½}=\frac{\partial}{\partial x}\left(D_H h\frac{\partial s}{\partial x}\right)_k+\frac{\partial}{\partial y}\left(D_H h\frac{\partial s}{\partial y}\right)_k+\left(\frac{J_z}{\rho}\right)\Bigg|_{k+½}^{k-½} \tag{12}$$

The notation $(\;)\big|_{k+½}^{k-½}$ represents the difference between interface values for a particular layer. Layer-averaged density $\rho_k$ has been substituted for $\rho_0$ in the denominator of the pressure, vertical stress and vertical salt flux terms. This substitution reduces any error caused by the Boussinesq approximation. The bottom and free surface boundary conditions are satisfied by defining $w_{km+½} = 0$, $(uw)_{½} = (uw)_{km+½} = 0$, $(vw)_{½} = (vw)_{km+½} = 0$, $(\tau_{xz}, \tau_{yz})_{½} = (\tau_{xs}, \tau_{ys})$, $(\tau_{xz}, \tau_{yz})_{km+½} = (\tau_{xb}, \tau_{yb})$, $(ws)_{½} = (ws)_{km+½} = 0$ and $(J_z)_{½} = (J_z)_{km+½} = 0$. The summation term in Equations 10 and 11 is omitted for a surface layer ($k$ = 1). These five three-dimensional governing equations are discretized using semi-implicit leapfrog and semi-implicit trapezoidal finite-differencing schemes.